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Task Compatibility of Manipulator Postures

Abstract

In performing a manipulation task, humans tend to adopt arm postures that most effectively utilize the motion and strength capabilities of the arm. Selecting arm postures that are compatible with the task requirements has become almost instinctive to humans. By mimicking this approach in robotic manipulation, we can exploit the full capability of a manipulator in performing a task. An index is proposed for measuring the compatibility of manipulator postures with respect to a generalized task description. The manipulator is viewed as a mechanical transformer, with joint space velocity and force as input and task space velocity and force as output. Optimization of the index corresponds to matching the velocity and force transmission characteristics to the task requirements. The applications of this index to manipulator redundancy utilization and workspace design are also discussed.

1. Introduction

Manipulation tasks are most conveniently perceived as a sequence of actions in Cartesian task coordinates. However, a task must ultimately be realized as a sequence of actions in the actuator coordinates, e.g., joint coordinates, of the manipulator. Motion or force in the task coordinates is invariably the result of motion or force in the actuator coordinates.

For a given posture of a manipulator, there are "preferred directions" for motion and for force exertion in task coordinates. There will be directions of motion in task coordinates that require minimum motion in actuator coordinates and directions of force exertion that require minimum loading on the actuators. In these respective directions, the range of achievable velocity and force is at a maximum, but the resolution, or accuracy, of control is at a minimum. Hence, we will distinguish optimal directions for *effecting* velocity and force from those for *controlling* them. The optimal directions for accurate control of velocity and force are those that require maximum velocity and force in actuator coordinates. In a recent work on the selection of grasp configurations for dexterous robot hands, Kerr and Roth (1986) suggested that a grasp should be selected to match the optimal directions of the hand with the task geometry. Although no quantitative measure was given for determining the optimality of grasp configurations, their work introduced the idea of matching a manipulator's posture to the task requirements.

We explored the idea of task-dependent postures in the context of redundant manipulator control (Chiu 1987). An index was proposed for measuring the compatibility of manipulator postures with respect to task requirements. Simulations for an anthropomorphic arm resulted in postures that agree with human experience. However, the proposed index was only applicable to limited types of tasks. In this paper, we extend the formulation of the index to incorporate a generalized task description. The utilization of this index in determining the optimal posture of redundant manipulators and its application to workspace design are also discussed.

2. The Velocity and Force Ellipsoids

For our purposes, it is appropriate to view the manipulator as a mechanical transformer with joint velocity and force as input and Cartesian velocity and force as output. The optimal direction for effecting velocity is the direction in which the transmission ratio of velocity is at a maximum. Conversely, the optimal direction for fine control of velocity is the direction in which the transmission ratio is at a minimum. The same concept applies to force.

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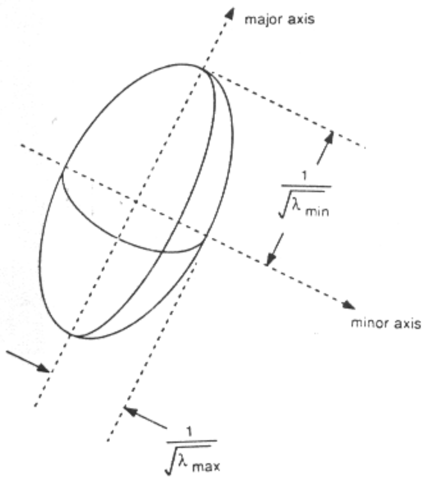


Fig. 1. Velocity ellipsoid.

2.1. The Velocity Ellipsoid

The velocity and force transmission characteristics of a manipulator at any posture can be represented geometrically as ellipsoids. Consider an n-degree-of-freedom manipulator with joint coordinates θ_i, $i = 1, 2, \ldots, n$, and a task described by m task coordinates x_j, $j = 1, 2, \ldots, m$ with $m \leq n$. Let the kinematic transformation from joint space to task space be given by

$$\mathbf{x} = \mathbf{x}(\boldsymbol{\theta})$$

where $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_n]^T$ and $\mathbf{x} = [x_1, x_2, \ldots, x_m]^T$ are the joint and task coordinate vectors. Differentiating with respect to time, we obtain

$$\dot{\mathbf{x}} = \mathbf{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \tag{1}$$

where $\mathbf{J}$ is the $m \times n$ Jacobian matrix, with elements given by $J_{i,j} = \partial x_i / \partial \theta_j$. From Eq. (1), we see that the Jacobian is simply a linear transformation that maps the joint velocity in R^n into a task velocity in R^m. The unit sphere in R^n defined by

$$\|\dot{\boldsymbol{\theta}}\|^2 = \dot{\theta}_1^2 + \dot{\theta}_2^2 + \cdots + \dot{\theta}_n^2 \leq 1$$

is mapped into an ellipsoid in R^m defined by

$$\dot{\mathbf{x}}^T(\mathbf{J}\mathbf{J}^T)^{-1}\dot{\mathbf{x}} \leq 1. \tag{2}$$

Yoshikawa (1984) called this the "manipulability ellipsoid," and proposed that kinematic redundancy should be used to maximize the volume of this ellipsoid. This is an effective means for singularity avoidance, since the volume of this ellipsoid is zero at singularities (where the achievable task velocity collapses into a lower dimension). The posture of various types of nonredundant manipulators has also been analyzed using the ellipsoid volume as a criterion (Yoshikawa 1985). For clarity, we will refer to this ellipsoid as the *velocity ellipsoid.*

The velocity ellipsoid is a useful tool for visualizing the velocity transmission characteristics of a manipulator at a specific posture. The principal axes of the velocity ellipsoid coincide with the eigenvectors of $(\mathbf{J}\mathbf{J}^T)^{-1}$, and the length of a principal axis is equal to the reciprocal of the square root of the corresponding eigenvalue (see Fig. 1). The optimal direction for effecting velocity is along the major axis of the ellipsoid, where the transmission ratio is at a maximum. Conversely, the velocity is most accurately controlled along the minor axis of the ellipsoid, where the transmission ratio is at a minimum.

2.2. The Force Ellipsoid

Analogous to the velocity ellipsoid, we can also define a force ellipsoid for describing the force transmission characteristics of a manipulator at a given posture. Forces in joint space and task space are mapped via the same Jacobian through the relation

$$\boldsymbol{\tau} = \mathbf{J}^T\mathbf{f} \tag{3}$$

where $\mathbf{f}$ is the force vector in task space and $\boldsymbol{\tau}$ is the joint torque vector. This relation can be derived from Eq. (1) by equating mechanical power in joint space with that in task space. That is,

$$\dot{\boldsymbol{\theta}}^T\boldsymbol{\tau} = \dot{\mathbf{x}}^T\mathbf{f} = \dot{\boldsymbol{\theta}}^T\mathbf{J}^T\mathbf{f}.$$

Using Eq. (3), we obtain

$$\tau^T\tau = \mathbf{f}^T(\mathbf{J}\mathbf{J}^T)\mathbf{f}.$$

Hence, the set of achievable force in R^m subject to the constraint $\|\tau\|^2 \leq 1$ is the ellipsoid defined by

$$\mathbf{f}^T(\mathbf{J}\mathbf{J}^T)\mathbf{f} \leq 1. \tag{4}$$

We will refer to this ellipsoid as the *force ellipsoid.* Analogous to the velocity ellipsoid, the principal axes of the force ellipsoid coincide with the eigenvectors of $(\mathbf{J}\mathbf{J}^T)$, and the length of a principal axis is equal to the reciprocal of the square root of the corresponding eigenvalue. This is effectively the same ellipsoid utilized by Asada and Youcef-Toumi (1984a) in the analysis of the power to force conversion characteristics of a direct-drive arm. The mean value of the lengths of the principal axes, which is basically also a measure of the volume, was used as a measure of the overall power conversion efficiency. This formed the basis for the design of a parallel drive mechanism which has a constant mean value independent of the posture (Asada and Youcef-Toumi 1984b).

2.3. Duality of Velocity and Force

Via Eqs. (2) and (4), the velocity ellipsoid is defined by the matrix $(\mathbf{J}\mathbf{J}^T)^{-1}$, and the force ellipsoid is defined by the matrix $(\mathbf{J}\mathbf{J}^T)$. It can be shown that $(\mathbf{J}\mathbf{J}^T)^{-1}$ and $(\mathbf{J}\mathbf{J}^T)$ have the same eigenvectors, and that the eigenvalues of $(\mathbf{J}\mathbf{J}^T)^{-1}$ are the reciprocals of the eigenvalues of $(\mathbf{J}\mathbf{J}^T)$. In other words, the principal axes of the velocity and force ellipsoids coincide, and the lengths of the axes are in inverse proportions (Yoshikawa 1985). This means the optimal direction for effecting velocity (maximum velocity transmission ratio) is also the optimal direction for controlling force (minimum force transmission ratio). Similarly, the optimal direction for effecting force is also the optimal direction for controlling velocity. This result can also be derived by applying Raleigh's principle to the velocity and force transformations (Kerr and Roth 1986).

The inverse force-velocity behavior is easily understood by our view of the manipulator as a mechanical transformer. Conservation of energy dictates that amplification in velocity transmission must invariably be accompanied by reduction in force transmission, and vice versa. From the point of view of control, this behavior also makes sense. We see that velocity is most accurately controlled in the direction where the manipulator can resist large disturbance forces, and force is most accurately controlled in the direction where the manipulator can quickly adapt its motion.

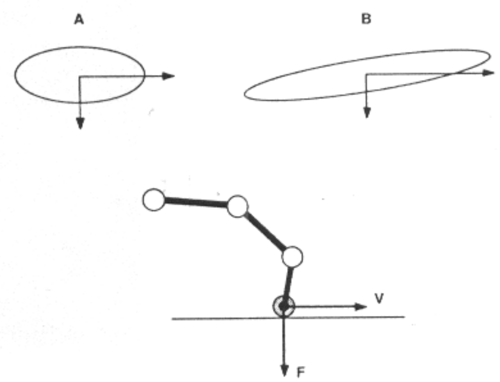


Fig. 2. Comparison of force ellipsoids for task compatibility.

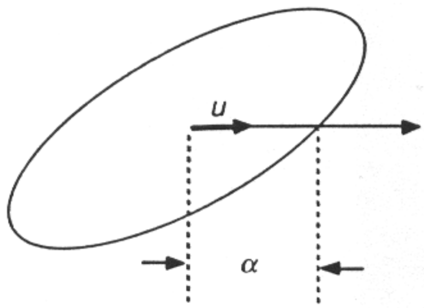
3. Task Compatibility

By varying the posture of a manipulator, we can change the shape and orientation of the velocity and force ellipsoids, and hence change the optimal directions for effecting or controlling velocity and force. The effective capability of a manipulator can be increased by adopting postures that align the optimal directions with the task directions. However, as we shall see, alignment of the optimal directions is not a sufficient criterion for determining the overall optimality of a posture.

3.1. Measure of Task Compatibility

Consider the contour-following example shown in Fig. 2. We wish to control force along the vertical direction

Fig. 3. Transmission ratio along a vector.



and velocity along the horizontal. The two ellipsoids shown are the force ellipsoids of a manipulator in two different postures. The transmission ratio along a particular direction is determined by the intersection of the directional vector with the surface of the ellipsoid. Although the optimal directions of ellipsoid A are aligned with the task directions, ellipsoid B is more "task compatible" because finer control of force and velocity can be achieved along the respective directions. This example shows that compatibility is not measured by alignment of optimal directions but by the actual transmission ratios along task directions.

3.2. Computation of Transmission Ratios

The transmission ratio along a particular direction is equal to the distance from the center to the surface of the ellipsoid along the directional vector. We will first consider the computation of the force transmission ratio. Let **u** denote the unit vector in the direction of interest, and let α be the distance along the vector **u** from the origin to the surface of the force ellipsoid (Fig. 3). The scalar α is then the force transmission ratio in the direction of **u**. Since $\alpha\mathbf{u}$ is a point on the ellipsoid, it must satisfy the equation (see Eq. (4))

$$(\alpha\mathbf{u})^{\mathrm{T}}(\mathbf{J}\mathbf{J}^{\mathrm{T}})(\alpha\mathbf{u}) = 1.$$

Hence, we have

$$\alpha = [\mathbf{u}^{\mathrm{T}}(\mathbf{J}\mathbf{J}^{\mathrm{T}})\mathbf{u}]^{-1/2}. \tag{5}$$

Through a similar derivation, the velocity transmission ratio β in the direction of **u** can be obtained as

$$\beta = [\mathbf{u}^{\mathrm{T}}(\mathbf{J}\mathbf{J}^{\mathrm{T}})^{-1}\mathbf{u}]^{-1/2}. \tag{6}$$

In the special case when **u** coincides with one of the principal axes of the ellipsoid, β can be computed as the reciprocal of α to avoid matrix inversion.

3.3. Compatibility Index

The transmission ratios represent the amplifications in force and velocity magnitude. The reciprocals of the transmission ratios represent the amplifications in control accuracy. The achievable manipulator performance in a task is maximized by optimizing these ratios in the task directions. Consider a task described by m task coordinates. We wish to control force along the direction vectors $\mathbf{u}_i$, $i = 1, 2, \ldots, l$, and to control velocity along $\mathbf{u}_j$, $j = l+1, l+2, \ldots, m$. Let the force transmission ratio in the direction of $\mathbf{u}_i$ be denoted by α_i, and let the velocity transmission ratio in the direction of $\mathbf{u}_j$ be denoted by β_j. We define the *compatibility index c* as

$$c = \sum_{i=1}^{l} w_i \alpha_i^{\pm 2} + \sum_{j=l+1}^{m} w_j \beta_j^{\mp 2}, \tag{7}$$

where the + sign is used for the directions in which the magnitude is of interest, and the − sign is used for the directions in which accuracy is of interest; w_i and w_j are weighting factors that indicate the relative magnitude or accuracy requirements in the respective task directions. We see that the compatibility index is a weighted sum of the squares of transmission ratios and squares of the reciprocals of transmission ratios. The problem of determining the optimal manipulator posture for a task can be restated as the problem of searching for the posture that maximizes this index.

Substituting Eqs. (5) and (6) into (7), the compatibility index is more explicitly defined by

$$c = \sum_{i=1}^{l} w_i\, [\mathbf{u}_i^{\mathrm{T}}(\mathbf{J}\mathbf{J}^{\mathrm{T}})\mathbf{u}_i]^{\pm 1} + \sum_{j=l+1}^{m} w_j [\mathbf{u}_j^{\mathrm{T}}(\mathbf{J}\mathbf{J}^{\mathrm{T}})^{-1}\mathbf{u}_j]^{\pm 1} \tag{8}$$

where the + sign is now used for the directions in which accuracy is of interest, and the − sign is used for the directions in which the magnitude is of interest.

An alternative measure of task compatibility can be obtained by extending the work of Li and Sastry (1987) on determining the optimal grasp locations on an object. Although their analysis is focused on the transformation of fingertip forces into net applied force on an object, the results can be directly extended to manipulator kinematics. In this approach, optimality is considered from the standpoint of maximizing the achievable range of force. The shape of a "task ellipsoid" is first determined from the relative force requirements along the task directions; i.e., the task ellipsoid represents a desired minimal force ellipsoid. The measure of optimality is then defined as the radius of the largest task ellipsoid that can be embedded in the actual force ellipsoid. Hence, optimizing this measure has the effect of simultaneously maximizing the volume of the force ellipsoid while coercing its shape to resemble that of the task ellipsoid. Although this approach can also be extended to include the definition of a task ellipsoid in the velocity domain to accommodate a hybrid velocity/force task description, the computational requirements are formidable.

4. Redundant Manipulator Control

Redundant manipulators have more degrees of freedom than that required for performing a task. Generally, there exists a continuum of postures that can achieve a given task coordinate position. Works on control of redundant manipulators have included the use of these extra degrees of freedom to minimize the energy in motion (Khatib 1983), to avoid joint variable limits (Liegeois 1977; Konstantinov et al. 1982; Hollerbach and Suh 1985), and to avoid singularity and obstacles (Yoshikawa 1984; Klein 1985). Dubey and Luh (1986) utilized redundancy to maximize either the velocity or force transmission ratio along the direction of the trajectory. The velocity transmission ratio is maximized if the manipulator is moving in free space; the force transmission ratio is maximized if the manipulator is interacting at low speed with external objects. In this section, we discuss the use of redundancy to maximize the task compatibility of a manipulator.

4.1. Optimization of Performance Criterion

The velocity of a redundant manipulator is mapped from a higher-dimension joint space to a lower-dimension task space via Eq. (1). The general solution to the inverse problem is given by

$$\dot{\theta} = \mathbf{J}^{+}\dot{\mathbf{x}} + (\mathbf{I} - \mathbf{J}^{+}\mathbf{J})\boldsymbol{\kappa}, \tag{9}$$

where $\mathbf{J}^{+}$ is the pseudoinverse of the Jacobian matrix $\mathbf{J}$ defined by

$$\mathbf{J}^{+} = \mathbf{J}^{\mathrm{T}}(\mathbf{J}\mathbf{J}^{\mathrm{T}})^{-1}$$

and $\boldsymbol{\kappa}$ is an arbitrary vector in R^{n}. The term $(\mathbf{I} - \mathbf{J}^{+}\mathbf{J})\boldsymbol{\kappa}$ corresponds to an internal motion of the joints that does not produce an endpoint motion. The goal here is to find a $\boldsymbol{\kappa}$ such that the internal motion will always move the manipulator toward a posture where the conpatibility index is at a maximum.

A solution for selecting $\boldsymbol{\kappa}$ to maximize performance criteria of the form $p(\theta)$ is given by Yoshikawa (1984). The internal motion will always increase $p(\boldsymbol{\theta})$ if

$$\boldsymbol{\kappa} = \frac{\partial \mathbf{p}}{\partial \theta} k, \tag{10}$$

where

$$\frac{\partial \mathbf{p}}{\partial \theta} = \left[\frac{\partial p}{\partial \theta_1}, \frac{\partial p}{\partial \theta_2}, \ldots, \frac{\partial p}{\partial \theta_n}\right]^{\mathrm{T}}$$

and k is a positive constant.

4.2. Simulation Results

Consider the planar 3-degree-of-freedom manipulator shown in Fig. 4. Given a required endpoint position,

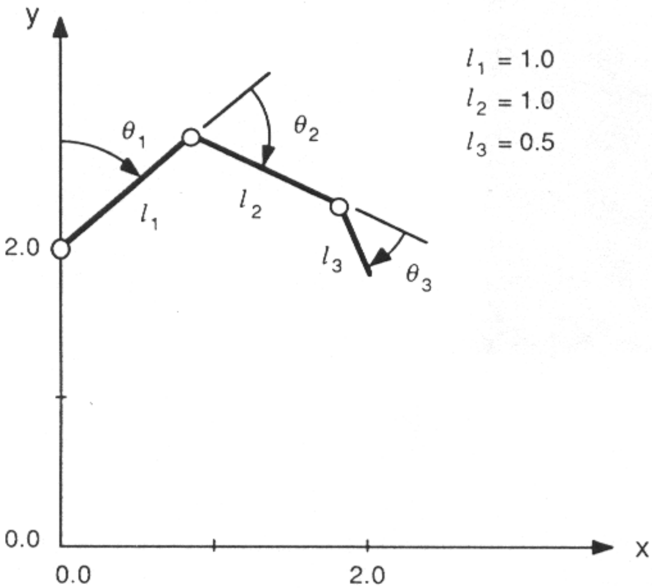


Fig. 4. Planar 3-degree-of-freedom manipulator.

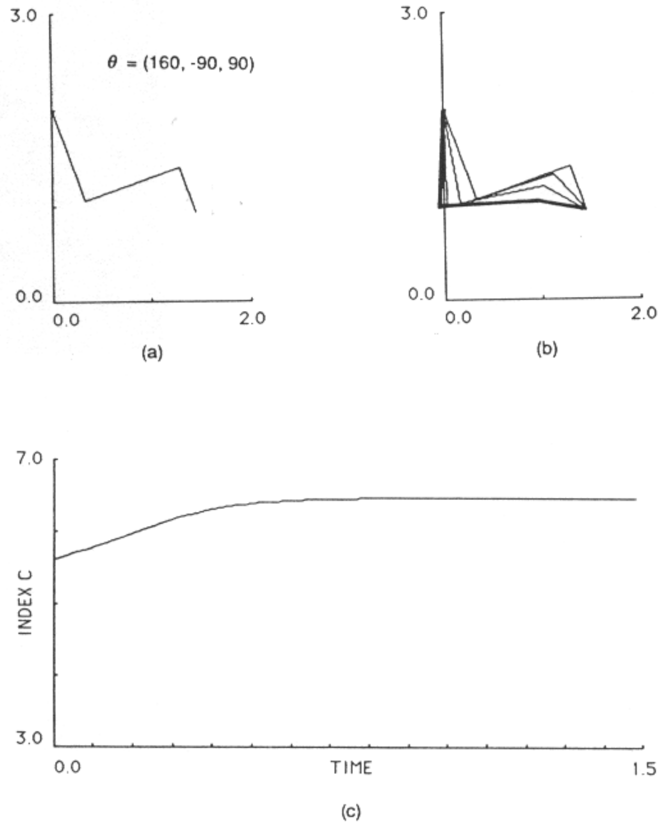


Fig. 5. Optimization of compatibility index. A. Initial posture. B. Evolution of posture. C. Compatibility index as a function of time.

we wish to find the posture that maximizes the compatibility index for accurate control of vertical force and horizontal velocity. From Eq. (8), the compatibility index for this task is

$$c = w_1[\mathbf{u}_1^{\mathrm{T}}(\mathbf{J}\mathbf{J}^{\mathrm{T}})\mathbf{u}_1] + w_2[\mathbf{u}_2^{\mathrm{T}}(\mathbf{J}\mathbf{J}^{\mathrm{T}})^{-1}\mathbf{u}_2],$$

where $\mathbf{u}_1 = [0 \quad 1]^{\mathrm{T}}$ and $\mathbf{u}_2 = [1 \quad 0]^{\mathrm{T}}$. The weighting factors w_1 and w_2 represent the relative importance of force control and velocity control, respectively. We will consider the case where $w_1 = w_2 = 1$. The Jacobian matrix of the manipulator is given by

$$\mathbf{J}(\boldsymbol{\theta}) = \begin{bmatrix} l_1c_1 + l_2c_{12} + l_3c_{123} & l_2c_{12} + l_3c_{123} & l_3c_{123} \\ -l_1s_1 - l_2s_{12} - l_3s_{123} & -l_2s_{12} - l_3s_{123} & -l_3s_{123} \end{bmatrix},$$

where $c_1 = \cos(\theta_1)$, $c_{12} = \cos(\theta_1 + \theta_2)$, $c_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$, $s_1 = \sin(\theta_1)$, $s_{12} = \sin(\theta_1 + \theta_2)$, and $s_{123} = \sin(\theta_1 + \theta_2 + \theta_3)$.

The initial posture of the manipulator is shown in Fig. 5A. Using the compatibility index as the performance criterion, we compute the joint motion that will maximize the index from Eqs. (9) and (10), using $k = 3$. The resulting evolution of the manipulator posture is shown in Fig. 5B, and the compatibility index as a function of time is shown in Fig. 5C. It is interesting to note that the best posture for fine control of vertical force and horizontal velocity resembles that of the human arm during writing.

Because the compatibility index is a nonlinear function of the variables, multiple maxima may exist. The proximity of the initial posture to these maxima will determine whether the posture will converge to the global maximum or a local maximum. Figure 6A shows an alternative initial posture that will place the endpoint at the same position as in Fig. 5A. The manipulator posture and the compatibility index during optimization are shown in Figs. 6B, C. We note that the final posture corresponds to a local maximum of the compatibility index, as evinced by the lower index value. Postures that correspond to local maxima of the compatibility index usually appear awkward, being uncommon in human experience. Utilizing heuristics to constrain the search is a subject for future work.

In the above examples, the search algorithm is based on analytical evaluation of the gradient of the compat-

Fig. 6. Optimization of compatibility index using alternative initial posture. A. Initial posture. B. Evolution of posture. C. Compatibility index as a function of time.

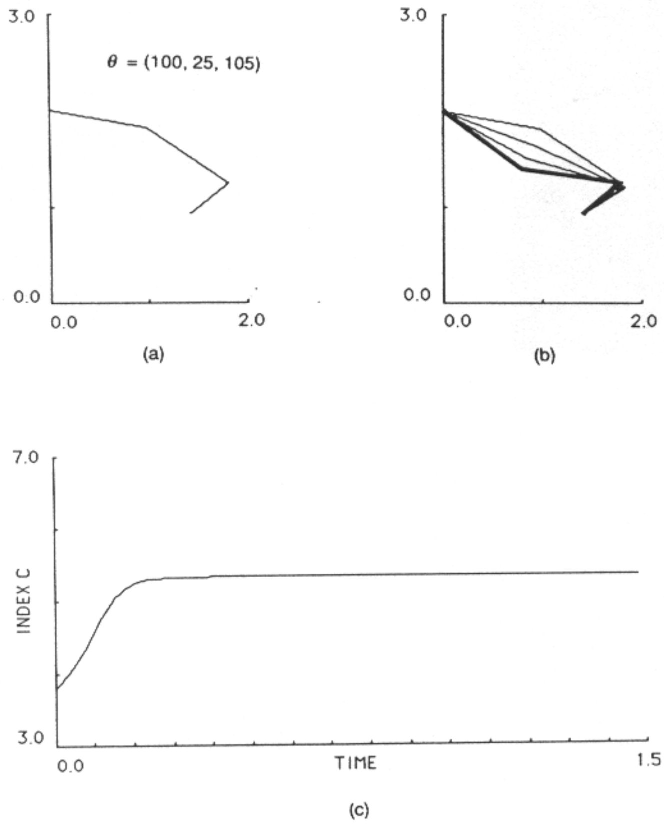
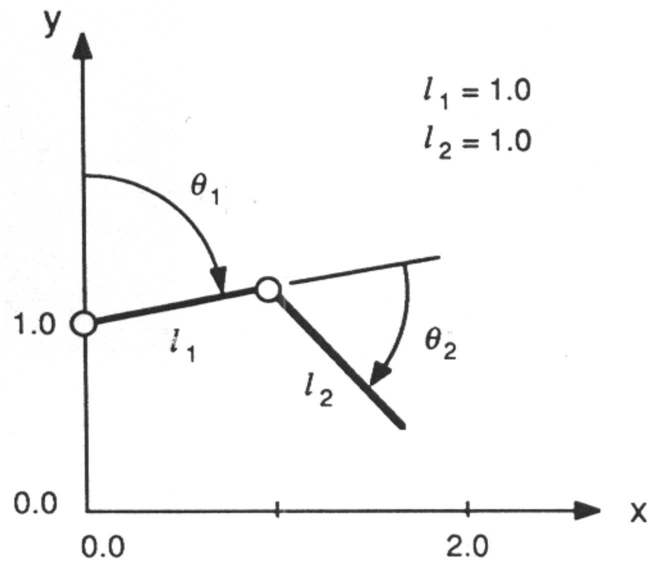


Fig. 7. Planar 2-degree-of-freedom manipulator.



ibility index. For applications to real-time control, time constraints may preclude the evaluation of complex expressions of the gradient. In this case, the gradient may need to be evaluated numerically in the form of difference equations.

5. Application to Workspace Design

For a nonredundant manipulator, the task coordinate position effectively determines the posture. It is then possible to compute the compatibility index as a function of the manipulator position in the task coordinates. The optimal position will correspond to the location in workspace where the task can be performed most effectively.

Consider the planar 2-degree-of-freedom manipulator shown in Fig. 7. We wish to find the optimal workspace location for a task that requires large vertical and horizontal force. The compatibility index for this task is

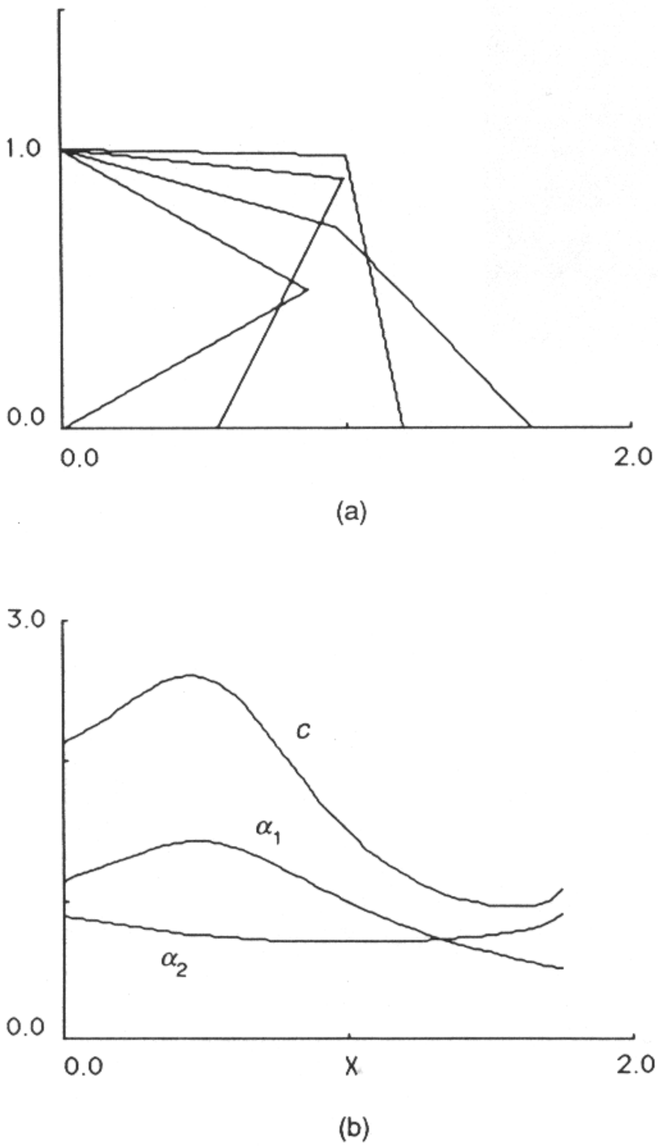
$$c = w_1[\mathbf{u}_1^T(\mathbf{J}\mathbf{J}^T)\mathbf{u}_1]^{-1} + w_2[\mathbf{u}_2^T(\mathbf{J}\mathbf{J}^T)\mathbf{u}_2]^{-1}$$

where $\mathbf{u}_1 = [0 \quad 1]^T$ and $\mathbf{u}_2 = [1 \quad 0]^T$. For simplicity, we will again only consider the case where $w_1 = w_2 = 1$. The Jacobian matrix of the manipulator is

$$\mathbf{J} = \begin{bmatrix} l_1c_1 + l_2c_{12} & l_2c_{12} \\ -l_1s_1 - l_2s_{12} & -l_2s_{12} \end{bmatrix}.$$

For $y = 0$, the compatibility index as a function of the manipulator x coordinate is shown in Fig. 8. The force transmission ratios α_1 and α_2 are also shown in the figure. Although the maximum of the compatibility index indicates the optimal location for a task, often certain force and velocity constraints must be satisfied. For example, there may be a minimum force requirement equivalent to the constraint $\alpha_2 \geqslant \alpha_{\min}$, where $\alpha_{\min}$ is a constant. The optimal location subject to a set of constraints can be easily determined from graphs similar to Fig. 8. An alternative approach is to adjust the weighting factors of the compatibility index until a maximum is found that satisfies the set of constraints.

Fig. 8. Workspace design A. Manipulator posture. B. Compatibility index as functions of the x coordinate.



6. Conclusion

The compatibility of manipulator postures with respect to task requirements has been studied in this paper. We viewed the manipulator as a mechanical transformer and focused on the velocity and force transmission ratios along the task directions. The optimization of the transmission ratios formed the basis for determining the posture most compatible with the task requirements.

An index has been proposed for measuring the compatibility of manipulator postures with respect to a generalized task description. The maximization of this index presents an effective way of utilizing manipulator redundancy. The index can also be applied to nonredundant manipulators as a criterion in workspace design.

The analysis presented in this paper is based only on kinematic considerations. However, the dynamics of a manipulator becomes important in many tasks. Further study is needed to examine the effects of arm dynamics on different aspects of task performance.

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References

Asada, H., and Youcef-Toumi, K. 1984a. Development of a direct-drive arm using high torque brushless motors. In *Robotics research: the first international symposium,* eds. M. Brady and R. Paul, Cambridge: MIT Press, pp. 583–599.

Asada, H., and Youcef-Toumi, K. 1984b. Analysis and design of a direct-drive arm with a five-bar-link parallel drive mechanism. *ASME J. Dyn. Sys. Meas. and Control* 106:225–230.

Chiu, S. 1987 (March, Raleigh, N.C.). Control of redundant manipulators for task compatibility. *Proc. IEEE Int. Conf. on Robotics and Automation,* pp. 1718–1724.

Dubey, R., and Luh, J. Y. S. 1986 (Dec., Anaheim). Performance measures and their improvement for redundant robots. *Proc. ASME Winter Annual Meeting,* DSC-Vol. 3, pp. 143–151.

Hollerbach, J., and Suh, K. C. 1985 (March, St. Louis).

Redundancy resolution of manipulators through torque optimization. *Proc. IEEE Int. Conf. on Robotics and Automation,* pp. 1016–1021.

Kerr, J., and Roth, B. 1986 (April, San Francisco). Special grasping configuration with dexterous hands. *Proc. IEEE Int. Conference on Robotics and Automation,* pp. 1361–1367.

Khatib, O. 1983 (Dec., New Delhi). Dynamic control of manipulators in operational space. *Proc. Sixth CISM-IFToMM Congress on Theory of Machines and Mechanisms.* New Delhi: Wiley, pp. 1128–1131.

Klein, C. 1985. Use of redundancy in the design of robotic systems. In *Robotics research: the second international symposium,* eds H. Hanafusa and H. Inoue, Cambridge: MIT Press, pp. 207–214.

Konstantinov, M., Patarinski, S., Zamanov, V., and Nenchev, D. 1982 (June, Paris). A contribution to the inverse kinematic problem for industrial robots. *Proc. 12th Int. Symp. on Industrial Robots,* pp. 459–468.

Li, Z., and Sastry, S. 1987 (March, Raleigh, N.C.) Task oriented optimal grasping by multifingered robot hands. *Proc. IEEE Int. Conf. on Robotics and Automation,* pp. 389–394.

Liegeois, A. 1977. Automatic supervisory control of the configuration and behavior of multibody mechanisms. *IEEE Trans. Systems, Man, Cybern.* SMC-7(12):868–871.

Yoshikawa, T. 1984. Analysis and control of robot manipulators with redundancy. In *Robotics research: the first international symposium,* eds. M. Brady and R. Paul, Cambridge: MIT Press, pp. 735–747.

Yoshikawa, T. 1985. Manipulability of robotic mechanisms. In *Robotics research: the second international symposium,* eds. H. Hanafusa and H. Inoue, Cambridge: MIT Press, pp. 439–446.